ADSP Final: Program 1 in Matlab

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- `Part1_SI_Filter.m`: Matlab code for part1
- `Part2_MSE_FIR.m`: Matlab code for part2
- `./image/`: Figures generated

Part 1: Step Invariance IIR Filter Design

Run Code

Input Options:

- `butter`: Generates a Butterworth low-pass filter.
- `cosine wave`: Generates a simple cosine wave filter with a cosine impulse response. [h(t) = \cos(2 \pi f_c t)]
- Could replace h(t) with other analog filters

The `fc` parameter represents the **cutoff frequencies** for Butterworth filters and the **center frequency** for cosine wave filters.

Key Steps

Line 50-60 breakdown the 3 key steps to generate digital filter from analog filter using step invariance:

```
%% Step 1: Calculate the convolution of h(t) and u(t)
h_au_t = int(h_t, t); % Integrate on h_t

%% Step 2: Perform sampling for h_au(t)
h_u_n = subs(h_au_t, t, n*T); % t -> n*T, from symbolic to discrete
h_u_n = double(h_u_n);

%% Step 3: Calculate h[n] from h_u[n] = h_u[n] - h_u[n-1]
% Since we set h_u[-1] = 0, h[0] = h_u[0]
h_n = [0, diff(h_u_n)];
```

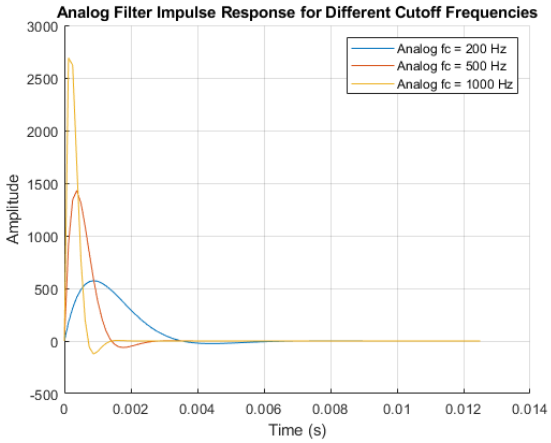
Pre-defined Parameters

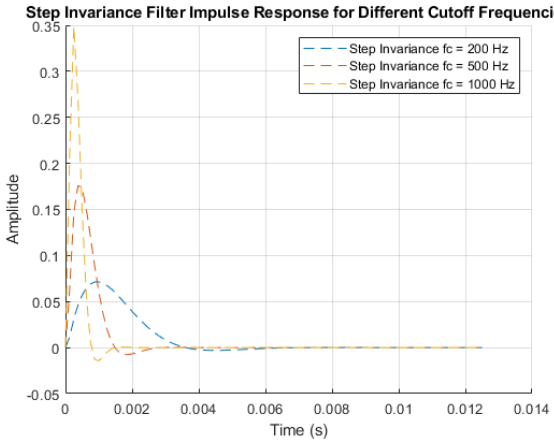
- `cutoff_frequencies`: A list of cutoff/center frequencies used for the filters, set to `[200, 500, 1000]` Hz.
- `fs`: Sampling frequency, set to `8000` Hz.
- `order`: The order of the Butterworth filter, set to `2`.
- `T`: Sampling period, calculated as `1 / fs`.
- `N`: Sampling length, set to `100`.
- `n`: Sample indices, ranging from `0` to `N`.

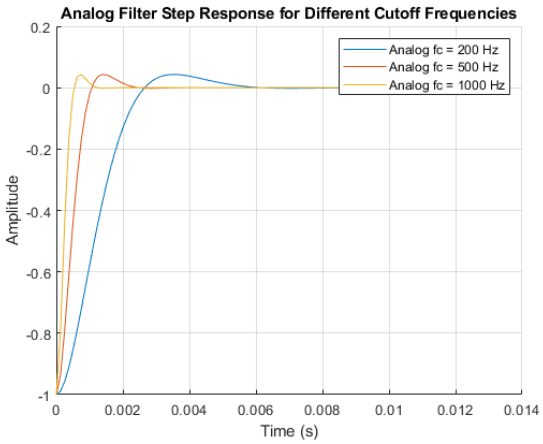
Run Results

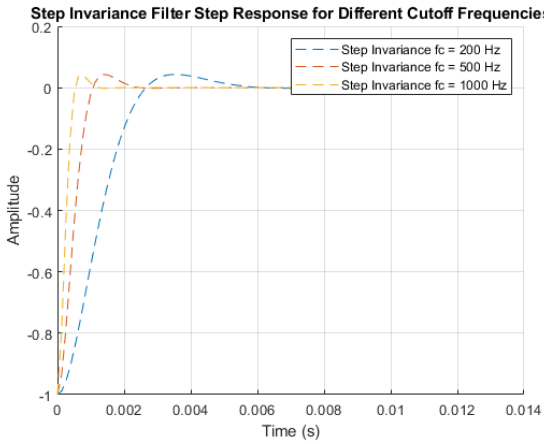
The script generates 6 figures for each case:

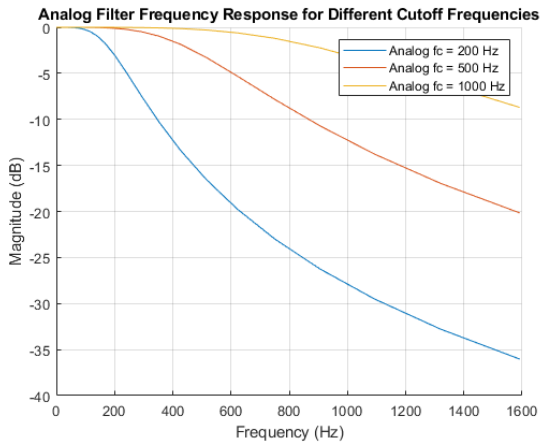
Case 1

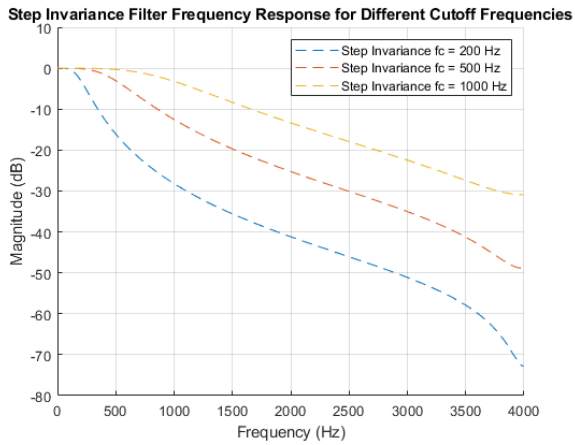




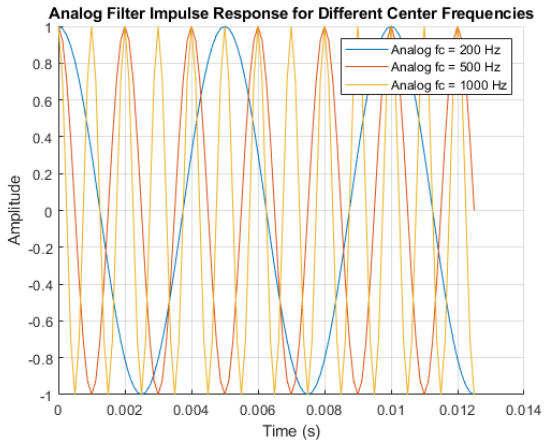


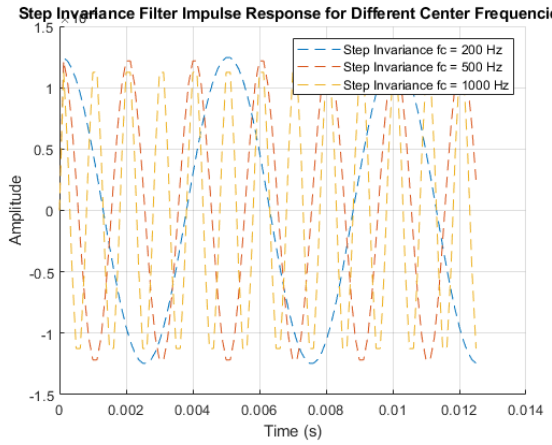


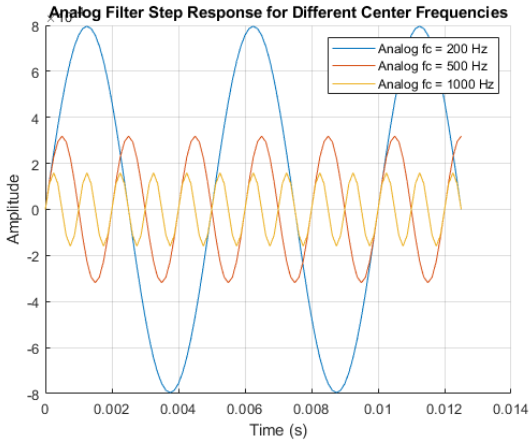


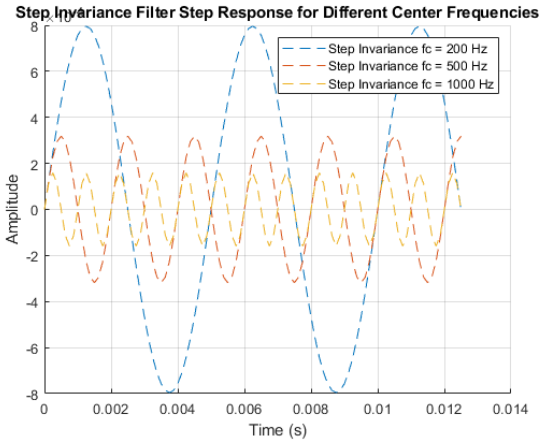


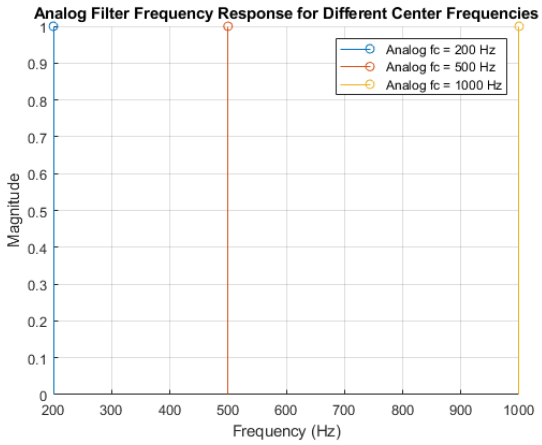
Case 2

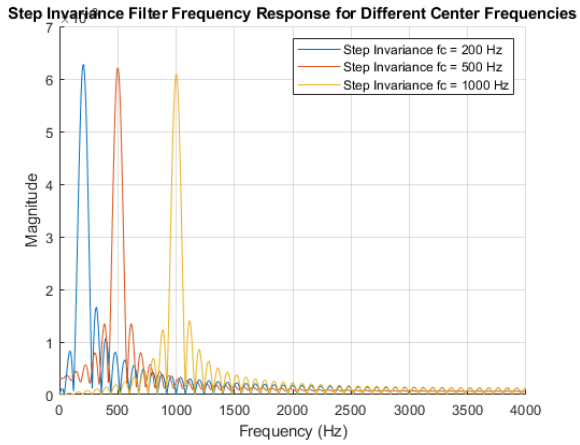












Step Response remains consistent after turning analog filter into IIR filter using step invariance method.

Part 2: MSE FIR Filter Design with Weights and Transition Bands

Run Code

Input Options:

- `filter_type`: Choose from 'low-pass', 'high-pass', 'band-pass', 'all-pass', 'notch'.
- `N`: Enter the filter length (default is 17).
- `fs`: Enter the sampling frequency (default is 6000 Hz).
- `passband_weight`: Enter the weight for the passband (default is 1).

- `stopband_weight`: Enter the weight for the stopband (default is 0.1).

Key Steps

Line 87-112 breakdown the key steps to generate MSE FIR filter:

```
%% Calculate s[n] by representing with (k+1) x (k+1) matrix operation
% Even with transition band, we already skip integrating F_0 to F_1 by
% making W(F) in this transition band to zero.
B = zeros(k+1, k+1);
c = zeros(k+1, 1);

for n = 0:k
    for tau = 0:k
        integrand_B = W .* cos(2*pi*tau*F) .* cos(2*pi*n*F);
        B(n+1, tau+1) = trapz(F, integrand_B);
    end
    integrand_c = W .* Hd .* cos(2*pi*n*F);
    c(n+1) = trapz(F, integrand_c);
end

s = B \ c;

%% Calculate h[n] from s[n]
n = 0:N-1;
h = zeros(size(n)); % h[0] ~ h[N-1]
h(k+1) = s(1); % h[k] = s[0]

for i = 1:k
    h(k + 1 + i) = s(i + 1) / 2;
    h(k + 1 - i) = s(i + 1) / 2;
end
```

Range of Passband and Stopband for Different Filter Types

- **Low-pass filter**:
  - Passband: (|F| < 0.225)
  - Stopband: (|F| \geq 0.225)
  - Transition band: (0.2 \leq |F| \leq 0.25)
- **High-pass filter**:
  - Passband: (|F| > 0.275)
  - Stopband: (|F| \leq 0.275)
  - Transition band: (0.25 \leq |F| \leq 0.3)
- **Band-pass filter**:
  - Passband: (0.225 \leq |F| \leq 0.275)
  - Stopband: (|F| < 0.225) or (|F| > 0.275)

- Transition band: (0.15 \leq |F| \leq 0.2) and (0.3 \leq |F| \leq 0.35)

- **All-pass filter**:
  - Passband: All frequencies
  - Stopband: None

- **Notch filter**:
  - Notch band centered at 0.25 with a width of 0.05

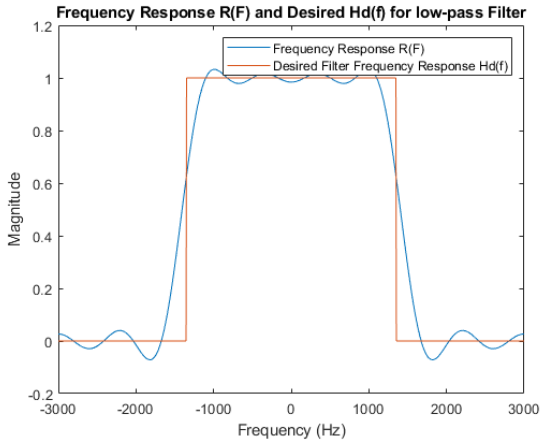
Weight Functions

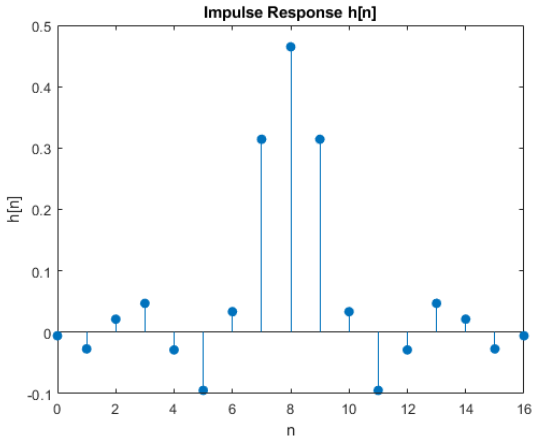
- `passband_weight` at passband
- `stopband_weight` at stopband
- `0` at transition band

Run Results

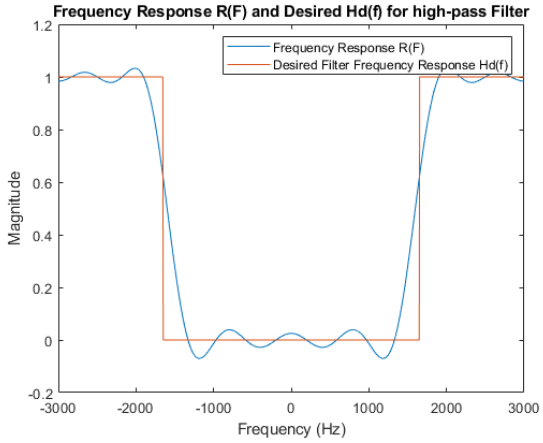
Containing 10 figures generated by 5 cases in default settings, the program also displays passband, stopband and MSE in the prompt after running:

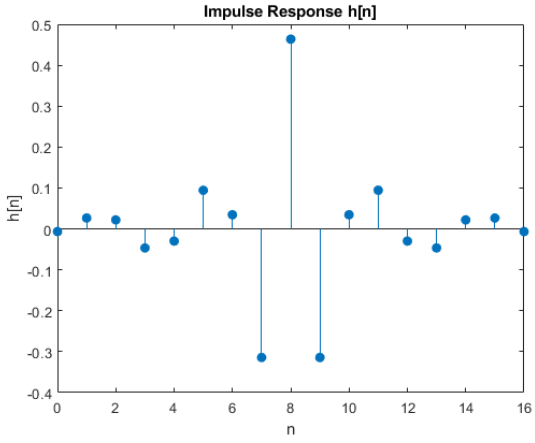
Low-pass Filter



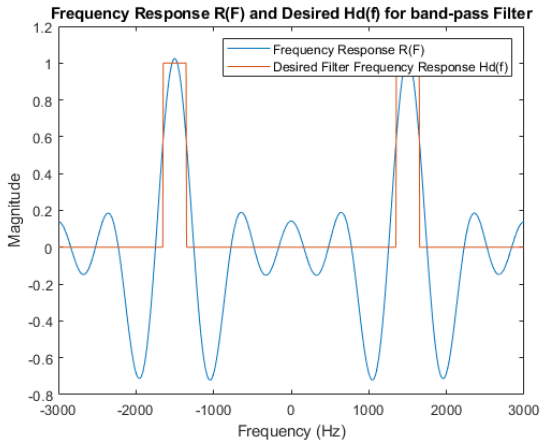


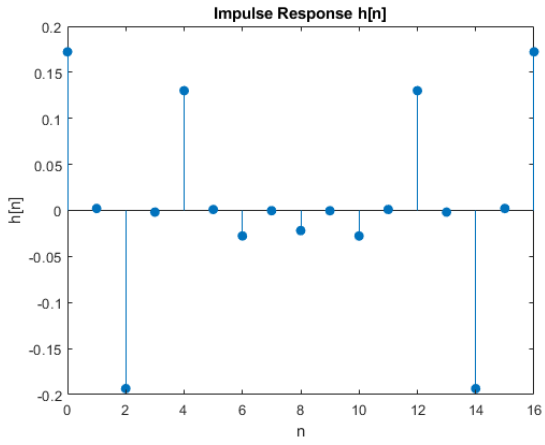
High-pass Filter



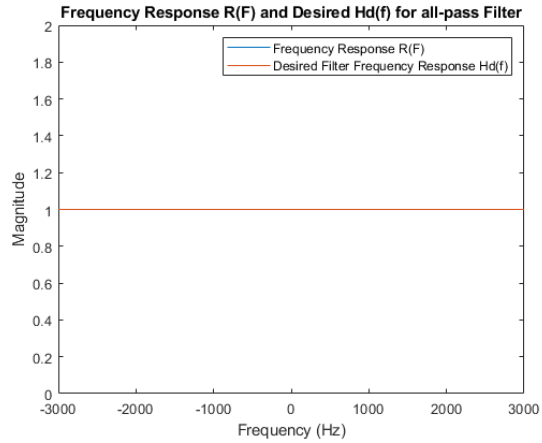


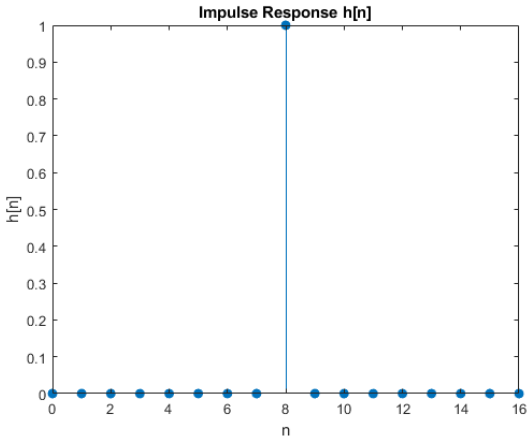
Band-pass Filter





All-pass Filter





Notch Filter